Recall that the transition amplitude for a scalar particle scattering off of a fixed Coulomb potential in de Sitter space is given by

$$T_{fi} = \delta_{fi} - i \delta_{l_1 l_2} \delta_{m m'} C_L C_{L'} \frac{2Qqa}{4\pi\epsilon_0} \int_{-\infty}^{\infty} \int_0^\pi \mathcal{F}(L, L', \chi)\chi_{L'}^*(t)\chi_L(t)\eta \cosh(t/a) \, d\chi \, dt. \quad (1)$$

By formulation of the scalar field solutions, the spatial and time dependence are completely decoupled. In order to derive a scattering selection rule in the angular momentum quantum number $l_1$, the spatial dependence can be expressed in only $l_1$-dependent terms as

$$T_{l_1 l_1} = \delta_{fi} - i \left[ (l_1' + 1) \frac{(l_1' + l_2 + 1)!}{(l_1' - l_2)!} \right]^{1/2} \left[ (l_1 + 1) \frac{(l_1 + l_2 + 1)!}{(l_1 - l_2)!} \right]^{1/2} \frac{\Gamma(l'_1 - l_2 + 1)}{\Gamma(l'_2 + 2)}$$

$$\times \int_0^\pi \left[ \sin \chi^{2(l_2 + 1)} \sum_{k=0}^{l_2} \frac{(-l_2)_k (l_1' - l_2 + 1)_k}{k!(l'_1 + 2)_k} \sin [(l_1 - l_2 + 2k + 1)\chi] \right.$$

$$\times \left\{ \sum_{k=0}^{l_2} \frac{(-l_2)_k (l_1' - l_2 + 1)_k}{k!(l'_1 + 2)_k} (l_1' - l_2 + 2k + 1) \cos [(l_1' - l_2 + 2k + 1)\chi] 
-(l_2 + 1) \cot \chi \sin [(l_1' - l_2 + 2k + 1)\chi] \right\} \, d\chi. \quad (2)$$

Removing terms that do not contribute to the $l_1$ selection rule (such as coefficient terms), this is reduced to

$$T_{l_1 l_1} = \delta_{fi} - i \int_0^\pi \sin \chi^{2(l_2 + 1)} \sum_{k=0}^{l_2} \sin [(l_1 - l_2 + 2k + 1)\chi] \left\{ \sum_{k=0}^{l_2} \cos [(l_1' - l_2 + 2k + 1)\chi] 
- \cot \chi \sin [(l_1' - l_2 + 2k + 1)\chi] \right\} \, d\chi. \quad (3)$$
It is important to notice that the trigonometric functions can be described in terms of
symmetry on the interval $\Delta = [0, \pi]$. For example, $\cot \chi$ is odd on $ \Delta$ and hence integrates
to zero. Also, $\sin n \chi$ is even on $\Delta$ for odd $n$ and odd on $\Delta$ for even $n$ while $\cos n \chi$ is odd
on $\Delta$ for odd $n$ and even on $\Delta$ for even $n$. Also, $\sin^{-2(l_2+1)} \chi$ is always even on $\Delta$. Thus,
when these sums in $T_{l_1 l_1'}$ are carried out, the integrals in equation 4 and 5 are produced. If
the $k$-index in the second sum is relabeled $k'$,

$$I_1 = \int_0^\pi \sin \chi^{-2(l_2+1)} \sin [(l_1 - l_2 + 2k + 1)\chi] \cos [(l_1' - l_2 + 2k' + 1)\chi]d\chi$$

and

$$I_2 = -\int_0^\pi \sin \chi^{-2(l_2+1)} \sin [(l_1 - l_2 + 2k + 1)\chi] \cot \chi \sin [(l_1' - l_2 + 2k' + 1)\chi]d\chi.$$  

Notice that in $I_2$, the integral over the product of sines vanishes unless $(l_1 - l_2 + 2k + 1) =
(l_1' - l_2 + 2k' + 1)$, or $(l_1 - l_1') = 2(k' - k)$. If this is the case, the function is even and the
product of this with the cot $\chi$ will cause $I_2$ to vanish. In the integral $I_1$, the product of the
sine and the cosine vanishes unless the functions are either both even or both odd. For both
of them to be even, $(l_1 - l_2 + 2k + 1)$ must be odd and $(l_1' - l_2 + 2k' + 1)$ must be even.
A similar criteria holds for both functions to be odd. In either case, it must be true that
$(l_1 - l_2 + 2k + 1) = (l_1' - l_2 + 2k')$, or $(l_1 - l_1') = 2(k' - k) - 1$. Thus, we have the scattering
selection rule for $l_1 \rightarrow l_1'$,

$$l_1 \rightarrow l_1 + 2n - 1 \geq 0, \ n \in \mathbb{Z} \quad \iff \quad T_{f_i} \neq 0,$$

along with the $l_2 \rightarrow l_2$ and $m \rightarrow m$ criteria.