

Cooper pairing in ultracold ^{40}K using Feshbach resonances

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We point out that the fermionic isotope ^{40}K is a likely candidate for the formation of Cooper pairs in an ultracold atomic gas. Specifically, in an optical trap that simultaneously traps the spin states $|9/2, -9/2\rangle$ and $|9/2, -7/2\rangle$, there exists a broad magnetic-field Feshbach resonance at $B = 196_{-21}^{+9}$ G that can provide the required strong attractive interaction between atoms. An additional resonance, at $B = 191_{10}^{+5}$ G, could generate p -wave pairing between identical $|9/2, -7/2\rangle$ atoms. A Cooper-paired degenerate Fermi gas could thus be constructed with existing ultracold-atom technology.

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Recently an ultracold gas of fermionic ^{40}K atoms was cooled to the quantum degenerate regime [1]. This achievement opens a new chapter in the story of ultracold matter, complementary to the Bose-Einstein condensation work that has been going on for over four years now. The degenerate Fermi gas (DFG) is expected to exhibit interesting behavior in its thermodynamics [2], collision dynamics [3], and scattering of light [4,5]. Perhaps the most intriguing prospect for the DFG is the potential to observe a pairing of the fermions, leading to a derived superfluid state, analogous to the Cooper pairing of electrons in a superconductor [6–10].

To make such a pairing work requires an effective attraction between colliding atom pairs in the gas. For bosons, an attractive interaction corresponds to a negative value of the s -wave scattering length. For fermions, however, the Pauli exclusion principle prohibits s -wave scattering of atoms in identical spin states. This leaves only p -wave collisions as a pairing mechanism, but the resulting interactions are energetically suppressed and are generally considered to give experimentally unattainable pairing transition temperatures [11]. On the other hand, a recent proposal has suggested that p -wave interactions may be enhanced by the application of very large dc electric fields, which could be generated by powerful CO_2 lasers [12].

A second possibility would be to use two different spin states of a fermionic atom, thus restoring s -wave collisions as a pairing mechanism. In this context ^6Li appears to be an attractive candidate [13], since it possesses a large negative s -wave scattering length $a_s = -2160a_0$, in units of the Bohr radius a_0 [14]. [In this paper we distinguish s - and p -wave scattering lengths with the subscripts s and p . To avoid confusion with standard notations, we indicate the singlet and triplet explicitly in superscripts, as in Eq. (3), below.] In this case the critical temperature for Cooper pairing is approximately [13]

$$T_c \sim \frac{E_F}{k_B} \exp\left(-\frac{\pi}{2k_F|a_s|}\right), \quad (1)$$

where E_F and k_F are the Fermi energy and momentum, respectively. As pointed out in Ref. [13], for experimentally

realizable Fermi energies $E_F/k_B \sim 600$ nK there would result $T_c \sim 15$ nK for ^6Li . Thus any alkali atom with a similarly large, negative scattering length should be a viable candidate for Cooper pairing.

The purpose of this paper is to consider the prospects for Cooper pairing in ^{40}K , in both s and p waves, based on a magnetic-field Feshbach resonance that can be used to tune its scattering length. This atom has been trapped and cooled in several labs [1,15]. The ability to tune scattering lengths resonantly using magnetic fields [16,17] is now a proven technology. To date, this resonant tuning was observed in Na [18], Rb [19,20], and Cs [21]. That these resonances are in fact useful tools for manipulation of ultracold gases has been amply demonstrated recently in an experiment that used them to Bose condense the otherwise uncondensable ^{85}Rb isotope [22].

Although the “required” scattering length to ensure formation of Cooper pairs will depend strongly on experimental circumstances, we can estimate a reasonable set of parameters using the guidelines laid out in Ref. [13]. A first requirement is that the resulting Cooper-paired state be mechanically stable, which for a two-component gas with number densities n_1 and n_2 requires [13]

$$n_1 n_2 a_s^6 \leq \left(\frac{\pi}{48}\right)^2. \quad (2)$$

When the scattering length violates this condition, two kinds of instability may occur: if $a_s < 0$, at least one component collapses into a dense, probably solid state; whereas if $a_s > 0$, the two components will phase separate [13]. Note that with a tunable a_s these instabilities can be probed experimentally in ^{40}K .

If we assume equal densities of $n_1 = n_2 = 10^{14}$ cm^{-3} , then Eq. (2) imposes the restriction $|a_s| < 1700a_0$. Conservatively, we will adopt a target value of $a_s = -1000a_0$ in the following. In this case, for a Fermi temperature of $T_F \sim 600$ nK (compare Ref. [1]), we would find a Cooper pairing temperature of $T_c \sim 25$ nK in ^{40}K . Moreover, we are interested in the stability of this T_c against variations in the magnetic-field strength. Let us require that T_c remain constant to within a small fraction, say 10%. Equation (1) then tells us that we

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must maintain a_s constant to within $\sim 3\%$. We will see below that this criterion should be relatively easy to meet for the resonance described.

To compute Feshbach resonances in ^{40}K , we employ the standard close-coupled Hamiltonian for ultracold alkali-atom scattering [23]. As usual, meaningful results can be obtained from this Hamiltonian only if it is fine tuned with the help of experimental data. In this case we will employ the constraints imposed by a recent analysis of photoassociation spectroscopy of the 0_g^- state of $^{39}\text{K}_2$ [24]. This analysis reveals a ^{39}K triplet scattering length (in a_0) of

$$a_s^{\text{triplet}}(39) = -17 - 0.045(C_6 - \bar{C}_6) \pm 25, \quad (3)$$

with $\bar{C}_6 = 3800$ atomic units [25]. This parametrization allows for an uncertainty in the C_6 coefficient that determines the long-range van der Waals attraction between the atoms. The experiment itself provides no direct information on the value of C_6 . The result in Eq. (3) is consistent with a complementary analysis of the 1_u state of ^{39}K , which gives $-60a_0 < a_s^{\text{triplet}}(39) < 15a_0$ [26].

Rescaling by the appropriate reduced mass, for ^{40}K Eq. (3) implies a nominal triplet scattering length of $a_s^{\text{triplet}}(40) = 176a_0$. This result is consistent with the values obtained in a direct collisional measurement in ^{40}K [27]. Finally, we take the singlet scattering length to be $a_s^{\text{singlet}}(40) = 105a_0$ [24,28]. This value is fairly well constrained by the existing data; moreover, the results of this paper depend only weakly on its exact value.

There remains the issue of the value of C_6 to employ in the calculations. The results of Marinescu *et al.* covered the fairly broad range $C_6 = 3800 \pm 200$ atomic units [25]. The accuracy of this result is limited by uncertainties in the atomic data used in the calculation. By contrast, a new high-precision calculation by Derevianko *et al.* predicted a much narrower range of $C_6 = 3987 \pm 15$ [29]. This improvement is largely due to Derevianko *et al.*'s accurate calculation of atomic structure, which freed them from experimental uncertainties. Their track record is impressive: for Na [30] and Rb [20], their predictions are within an experimental uncertainty of inferred values of C_6 . This result lends credence to their value of C_6 for potassium, which we will adopt here. In this case the largest uncertainty in potassium scattering lengths arise from the ± 25 in Eq. (3), rather than from C_6 . Taking this uncertainty into account, and rescaling the mass, the ^{40}K triplet s -wave scattering length is given by $a_s^{\text{triplet}}(40) = 176_{-27}^{+77}a_0$.

Perhaps the most appealing candidate spin states in which to seek a Feshbach resonance would be the magnetically trappable states $|fm\rangle = |9/2, 9/2\rangle$ and $|9/2, 7/2\rangle$, which are already trapped in the JILA experiment [1]. However, as reported in Ref. [28], no such resonance exists. There may be resonances for nearby spin states, but these should be very narrow ($\Delta B \ll 1$ G) and probably not useful for Cooper pairing.

There is, however, a broad resonance in collisions between the states $|9/2, -9/2\rangle$ and $|9/2, -7/2\rangle$, as illustrated in Fig. 1. This resonance, lying between 175 and 205 G, is

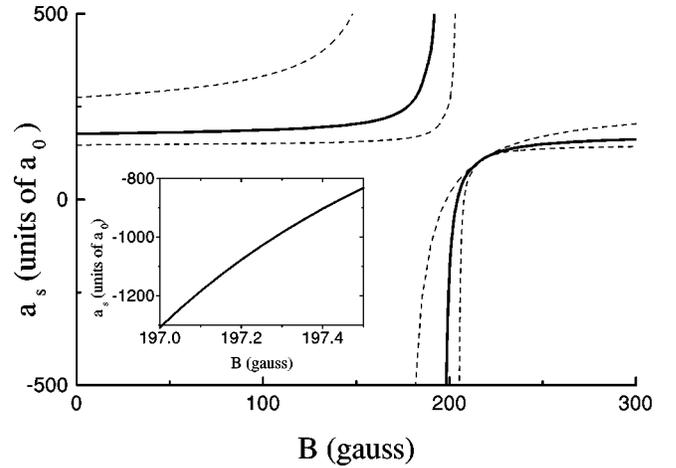


FIG. 1. Variation with magnetic field of the s -wave scattering length a_s for $|9/2, -9/2\rangle + |9/2, -7/2\rangle$ collisions of ^{40}K . The heavy line shows the nominal case, where $a_s^{\text{triplet}}(40) = 176a_0$, while the dotted lines indicate the uncertainty of the resonance's position through the uncertainty in $a_s^{\text{triplet}}(40)$. The inset shows the nominal case in the vicinity of $a_s = -1000a_0$.

easily accessible experimentally. Moreover, its broad width implies that the scattering length can be tuned quite accurately. The inset to Fig. 1 focuses on the region near $a_s = -1000a_0$. To maintain this value of the scattering length to within 3% (i.e., to maintain T_c constant to within 10%, as discussed above) would require holding B steady to within ~ 0.1 G. Since the two states are strong-field seekers, they cannot be trapped in the usual magnetic traps that have traditionally been used for Bose-Einstein condensation studies. Nevertheless, the two states could be held in an optical trap. These traps have recently attained great stability, with lifetimes exceeding 300 sec [31]. Note also that an optical trap ensures that the magnetic field can be made uniform across the entire trap, so that all atoms would experience the same pairing interaction. Evaporative cooling may be possible in these traps as well [32,33].

These particular spin states are also appealing in terms of their stability against collisional losses. At the ultralow temperatures of interest here, p -wave collisions are strongly suppressed, meaning that there are virtually no losses due to collisions between atoms in the same spin state. Inelastic collisions that produce $|9/2, -5/2\rangle$ states are also energetically forbidden, since the energy of this state lies 2.3 mK higher in energy than the $|9/2, -7/2\rangle$ state at the magnetic fields considered. There would then remain only the collision process

$$|9/2, -9/2\rangle + |9/2, -7/2\rangle \rightarrow |9/2, -9/2\rangle + |9/2, -9/2\rangle. \quad (4)$$

This collision cannot occur in a spin-exchange process, which must conserve the sum $m_a + m_b = -8$ of the magnetic quantum numbers. Nor can it proceed by the spin-spin dipolar interaction [34]. This is because an incident s wave can only couple to a d -wave final state in this processes, but d

waves are forbidden for identical final spin states. Thus the mixture we envision is virtually immune to two-body loss processes.

This leaves us with the possibility for three-body loss processes, where two bodies recombine into a molecule, with the other carrying away the binding energy. These processes can generally contribute to heating, trap loss, or contamination with unwanted molecular states. They have been observed to exert a strong influence on Bose-Einstein condensates, especially near Feshbach resonances [18]. The age of quantitative calculation of three-body recombination has just begun [35]. Nevertheless, we can argue that these losses, too, are suppressed in this system. Roughly this is because any three-body collision in a two-component Fermi gas must involve two identical atoms. Again invoking the exclusion principle, these atoms must have a nonzero relative angular momentum, which effectively keeps them apart, suppressing the collision. Following the more careful hyperspherical treatment of the type in Ref. [35], this would lead to a threshold law where the three-body recombination rate vanishes at low E as $E^{1/2}$, in contrast to the E -independent rate expected for bosons.

Finally, we return to the subject of possible p -wave Cooper pairing, similar to that envisioned in Ref. [12]. Non- s -wave pairing is already known in superconductors and in superfluid ^3He . However, the ability to produce this pairing in a dilute, weakly interacting atomic gas, and moreover to control the strength of coupling, would enable a detailed experimental and theoretical study, as was already the case for dilute Bose condensates. In this case the pairing temperature, analogous to Eq. (1), is given by

$$T_c \sim \frac{E_F}{k_B} \exp\left(-\frac{\pi}{2(k_F|a_p|)^3}\right). \quad (5)$$

Here a_p stands for the “ p -wave scattering length,” defined by

$$a_p^3 = \lim_{k \rightarrow 0} -\frac{\delta_p(k)}{k^3}, \quad (6)$$

where $\delta_p(k)$ is the p -wave scattering phase shift and k is the wave number. The cubic dependence on a_p of the exponential in Eq. (5) places more severe restrictions on a_p than in the s -wave case. For example, for $T_F = 600$ nK, setting $a_p = -1000a_0$ in Eq. (5) would yield a critical temperature of only $T_c = 0.002$ nK, whereas for $a_p = -1500$ we would obtain $T_c = 14$ nK. In the latter case, if we again require that T_c be constant to within 10%, we find that a_p must be constant to within 1%.

For ^{40}K the naturally occurring value of the triplet p -wave scattering length is $a_p^{\text{triplet}}(40) = -100a_0$, which is far too small to be of use. Fortunately, there are Feshbach resonances in this case, too. Generally speaking, these resonances lie at approximately the same values of the magnetic field as the s -wave resonances, since each s -wave bound state that can resonate is accompanied by a p -wave bound state at a nearby energy. For example, in p -wave collisions of

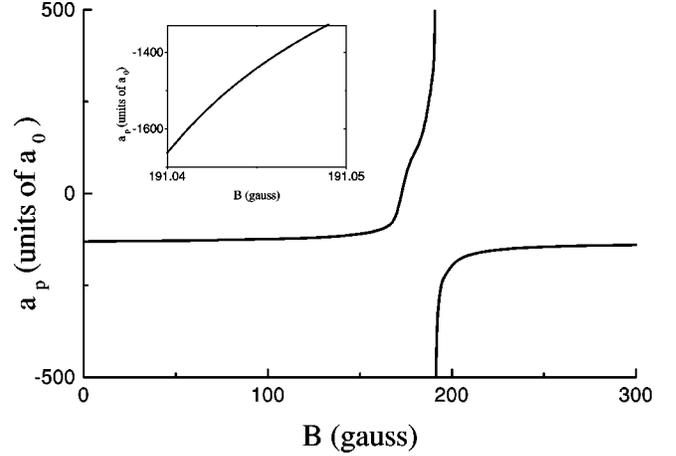


FIG. 2. Variation with magnetic field of the p -wave scattering length a_p for $|9/2, -7/2\rangle + |9/2, -7/2\rangle$ collisions of ^{40}K . Shown is the nominal case, where $a_p^{\text{triplet}}(40) = -100a_0$. The inset focuses on the variation of a_p with B in the vicinity of $a_p = -1500a_0$.

$|9/2, 7/2\rangle$ and $|9/2, 5/2\rangle$, there are extremely narrow resonances, as for the s -wave case.

We therefore again seek resonances in states with negative values of m_f . In particular, for collisions in a gas of pure $|9/2, -7/2\rangle$ atoms, we find a fairly broad resonance at a position of $B = 191_{-10}^{+5}$ G, as illustrated in Fig. 2. This resonance has nearly the same shape as the familiar s -wave resonances, but with an additional inflection when $a_p \sim 0$, arising from the cube-root dependence of a_p on δ_p . This resonance is also somewhat narrower in a magnetic field than the s -wave resonance reported above. In this case, holding $a_p = -1500a_0$ constant to within 1% requires holding the magnetic field constant to within perhaps 0.001 G.

In a given experiment the desired values of scattering lengths may differ from the sample values we have considered. In this case, it is useful to present approximate fitting formulas for the resonances, computed for the nominal interaction potentials. For s waves, this fit is

$$a_s \approx 164 - \frac{1260}{(B - 196.2)}. \quad (7)$$

The p -wave fit, for $a_p < 0$ and very near resonance, is

$$a_p \approx -600 - \frac{21}{(B - 191.02)}. \quad (8)$$

In each case, the scattering length is in a_0 and the field B is in G.

In conclusion, these magnetic-field Feshbach resonances make possible a variable Cooper-pairing interaction in ultracold ^{40}K gases. Such interactions will enable detailed studies of s - and p -wave superfluid states, including their instabilities, at a level not possible before. Significantly, to implement these resonances requires no technology beyond what is currently available.

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